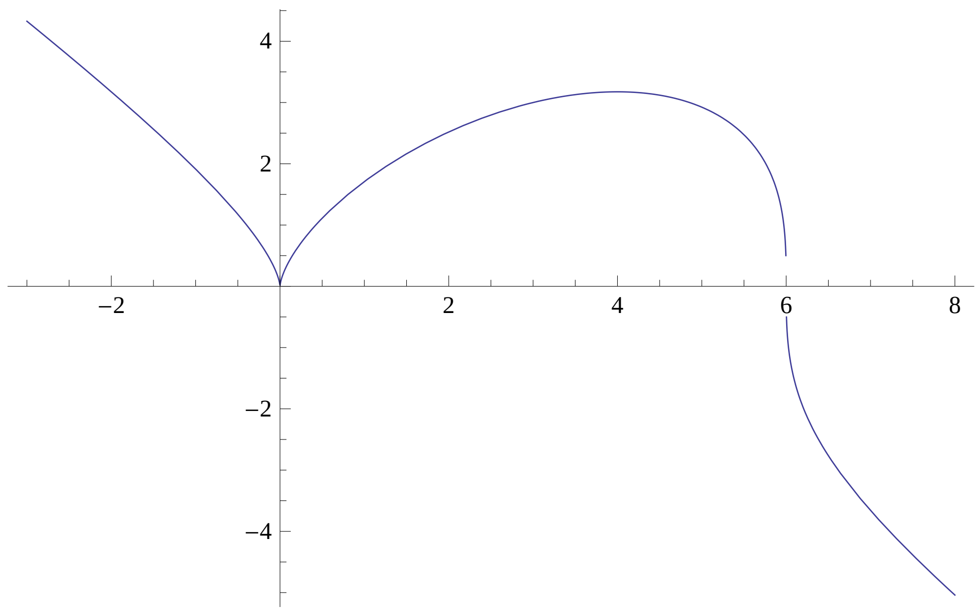
Ejemplo de Función con cúspides y tangentes verticales

MATE 3151

10 de octubre de 2013



$$\boxed{f(x) \;=\; x^{2/3}(6-x)^{1/3}}$$

Dado que $f(x) = x^{2/3}(6-x)^{1/3}$, entonces,

$$
\boxed{
\begin{aligned}
f'(x) &= x^{2/3}\left(\frac{1}{3}(6-x)^{-2/3}(-1)\right) + \frac{2}{3}x^{-1/3}\,(6-x)^{1/3} \\
&= \frac{-x^{2/3}}{3(6-x)^{2/3}} + \frac{2(6-x)^{1/3}}{3x^{1/3}} \\
&= \frac{-x^{2/3}\cdot x^{1/3} + 2(6-x)^{1/3}(6-x)^{2/3}}{3x^{1/3}(6-x)^{2/3}} \\
&= \frac{-x + 2(6-x)}{3x^{1/3}(6-x)^{2/3}} \\
&= \frac{-x + 12 - 2x}{3x^{1/3}(6-x)^{2/3}} \\
&= \frac{12 - 3x}{3x^{1/3}(6-x)^{2/3}} \\
&= \frac{3(4-x)}{3x^{1/3}(6-x)^{2/3}} \\
&= \frac{(4-x)}{x^{1/3}(6-x)^{2/3}}
\end{aligned}
}
$$

Dado que $f'(x) = \dfrac{(4-x)}{x^{1/3}(6-x)^{2/3}}$, entonces,

$$
\begin{aligned}
f''(x) &= \frac{x^{1/3}(6-x)^{2/3}(-1) - (4-x)\left[x^{1/3}\cdot\frac{2}{3}(6-x)^{-1/3}(-1) + \frac{1}{3}x^{-2/3}\cdot(6-x)^{2/3}\right]}{\left(x^{1/3}(6-x)^{2/3}\right)^2} \\
&= \frac{x^{1/3}(6-x)^{2/3}(-1) - (4-x)\left[x^{1/3}\cdot\frac{2}{3}(6-x)^{-1/3}(-1) + \frac{1}{3}x^{-2/3}\cdot(6-x)^{2/3}\right]}{x^{2/3}(6-x)^{4/3}} \cdot \frac{3x^{2/3}(6-x)^{1/3}}{3x^{2/3}(6-x)^{1/3}} \\
&= \frac{3x(6-x)(-1) - (4-x)\left[x\cdot 2(-1) + (6-x)\right]}{3x^{4/3}(6-x)^{5/3}} \\
&= \frac{3x(x-6) + (x-4)(6-3x)}{3x^{4/3}(6-x)^{5/3}} \\
&= \frac{3x^2 - 18x + 6x - 3x^2 - 24 + 12x}{3x^{4/3}(6-x)^{5/3}} \\
&= \frac{-24}{3x^{4/3}(6-x)^{5/3}} \\
&= \frac{-8}{x^{4/3}(6-x)^{5/3}}
\end{aligned}
$$