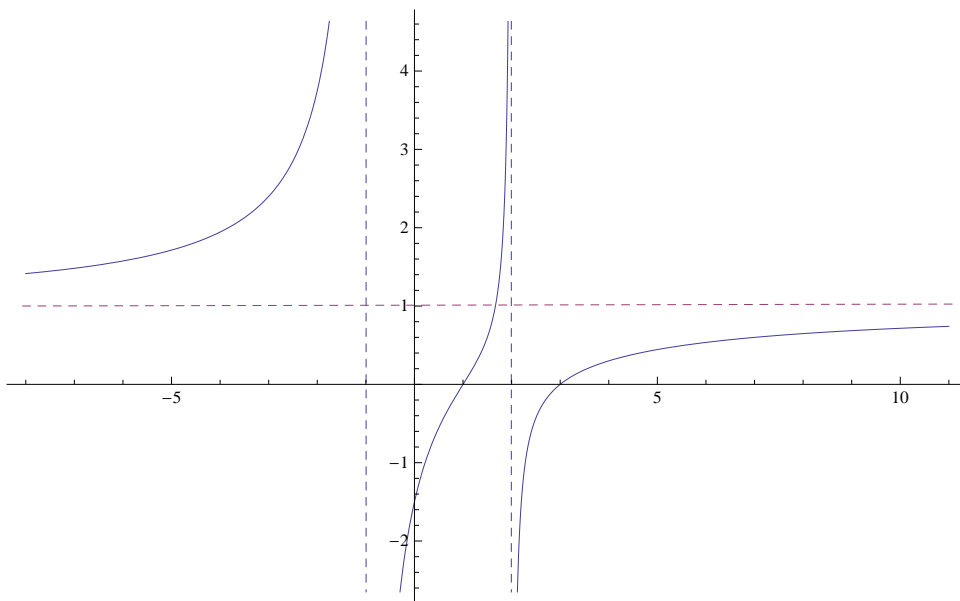# Ejemplo de Función Racional $\frac{P(x)}{Q(x)}$

MATE 3151

10 de octubre de 2013

$$f(x) \;=\; \frac{(x-1)(x-3)}{(x+1)(x-2)} = \frac{x^2-4x+3}{x^2-x-2}$$

$$f'(x) = \frac{(x^2 - x - 2)(2x - 4) - (x^2 - 4x + 3)(2x - 1)}{(x + 1)^2(x - 2)^2}$$

$$= \frac{2x^3 - 2x^2 - 4x - 4x^2 + 4x + 8 - (2x^3 - 8x^2 + 6x - x^2 + 4x - 3)}{(x + 1)^2(x - 2)^2}$$

$$= \frac{3x^2 - 10x + 11}{(x + 1)^2(x - 2)^2}$$

$$f''(x) = \frac{(x + 1)^2(x - 2)^2(6x - 10) - \left(3x^2 - 10x + 11\right)\left[(x + 1)^2 2(x - 2) + 2(x + 1)(x - 2)^2\right]}{(x + 1)^4(x - 2)^4}$$

$$= \frac{(x + 1)(x - 2)\left[(x + 1)(x - 2)(6x - 10) - \left(3x^2 - 10x + 11\right)\{2(x + 1) + 2(x - 2)\}\right]}{(x + 1)^4(x - 2)^4}$$

$$= \frac{(x^2 - x - 2)(6x - 10) - \left(3x^2 - 10x + 11\right)\{2x + 2 + 2x - 4\}}{(x + 1)^3(x - 2)^3}$$

$$= \frac{6x^3 - 6x^2 - 12x - 10x^2 + 10x + 20 - \left(3x^2 - 10x + 11\right)\{4x - 2\}}{(x + 1)^3(x - 2)^3}$$

$$= \frac{6x^3 - 6x^2 - 12x - 10x^2 + 10x + 20 - \left(12x^3 - 40x^2 + 44x - 6x^2 + 20x - 22\right)}{(x + 1)^3(x - 2)^3}$$

$$= \frac{-6x^3 + 30x^2 - 66x + 42}{(x + 1)^3(x - 2)^3}$$

$$= \frac{(x - 1)\left[-6x^2 + 24x - 42\right]}{(x + 1)^3(x - 2)^3}$$